

## Dose Distribution of Obliquely Incident Electron Beams using the Monte Carlo Method

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### Introduction

Mycosis fungoides is a T-cell lymphoma which involves the skin and lymph nodes and will eventually spread to the visceral organs causing death. Treatment with low energy electrons at an extended source distance providing a large radiation field with energy deposition near the skin surface is indicated.

Knowledge of the distribution of dose in a water equivalent phantom is required for accurate planning of electron therapies with a linear accelerator. Where radiation is applied to the whole body, such as is the case in the treatment of mycosis fungoides, obliquely incident particles contribute significantly and energy is deposited closer to the surface leading to an increase in the peak dose. The superficiality of the radiation makes the measurement of dose problematic at non-normal incidence so the prediction of these depth-dose distributions is a desirable compliment to measurements.

Computationally intensive methods such as the Monte Carlo (MC) method are necessary to accurately model the effects of scatter of electrons in the megavoltage energy range. The higher orders of correction provided by the standard MC method are inaccessible to most radiotherapy departments because it demands lengthy calculations including the pre-computation of an accurate phase-space description of the beam. More typically, a standard measurement of a reference beam provides the first approximation to the dose in

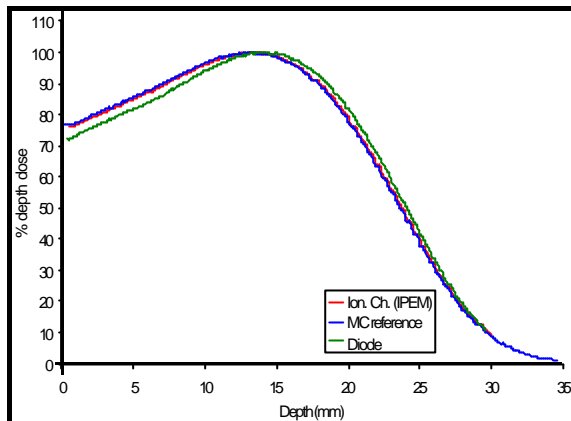
the patient and various geometry based corrections are used.

With the MC method it is not obvious how such measurements can be incorporated into the calculation. However, a calculation can be forced to converge on a measured reference distribution, via sampling from a prior such as the energy spectrum of the beam. The successive approximations to the reference distribution form a Markov chain [1] and concurrent estimations of the dose distribution in the patient can be determined via correlated sampling [2]. The technique is therefore a perturbation method and enables the efficient computation of dose distributions in circumstances where the treatment represents a small variation on a reference set of measurement that can be performed with accuracy [3].

This paper introduces the Markov chain Monte Carlo (MCMC) method, shows how it can be used to predict perturbed dose distributions from broad-parallel electron beams and presents calculations that have been made over a range of angles of incidence. The distributions are compared to measurements performed with a p-type silicon diode.

### Method

The reference distribution has been measured for a 6 MeV broad beam normally incident on a water phantom placed at 1 m from the source using an NACP parallel plate ionisation chamber.

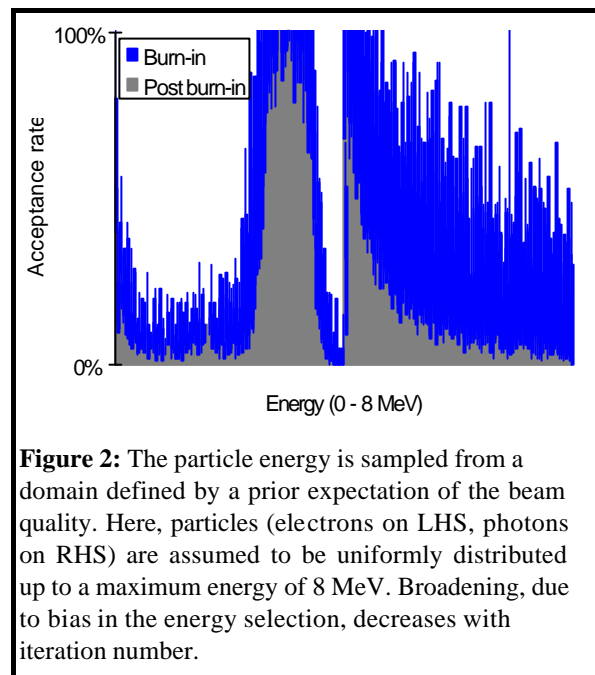


**Figure 1:** A NACP parallel plate ionisation chamber has been used to measure the reference dose distribution as a function of depth and has been corrected for changes in the mass-collision stopping power ratio of air to water with depth using the IPEM 1997 protocol. The mean energy at the surface is 5.395 MeV. Here, the silicon diode readings have been corrected using ICRU-37 tabulations of restricted stopping powers ( $\Delta = 10$  keV).

The Markov chain Monte Carlo method can be described as follows. For each iterative step,  $i$ , the probability of a distribution,  $D_i$ , occurring, given a set of measurements,  $D_0$ , denoted,  $P(D_i | D_0)$  is maximised. In practice, it is easier to compute a probability of the form,  $P(D_0 | D_i)$ , which is proportional to  $P(D_i | D_0)$  by Bayes' theorem, as  $D_i$  represents a Poisson distributed estimate of the solution and is computed via conventional Monte Carlo transport. Particle showers are computed using the code system Electron Gamma Shower version 4 (EGS4) [4].

Bayesian sampling introduces a bias into the calculation which causes a broadening in the spectrum. These are particles that through random variation appear to match the measured distribution but have variant scatter properties that lead to errors in the perturbation distribution. The error diminishes with successive iterations by increasing the number of particle histories per iteration and a "burn in" period is necessary to exclude large biases found in the earliest estimates (Figure 2).

In practice, two sets of particles are transported through geometries which are specific to the reference and treatment beams. These may be correlated if the local environments of the particles are generally equivalent. The ratio of the doses computed in each of these circumstances is a perturbation function that when multiplied by the measured distribution gives the absolute dose in the treatment beam. The detectors are assumed to respond in the manner of a Bragg-Gray cavity so mass-collision stopping power corrections for the detectors are computed concurrently using a look up table of restricted stopping powers of water, air and silicon that are used to weight individual energy depositions.



**Figure 2:** The particle energy is sampled from a domain defined by a prior expectation of the beam quality. Here, particles (electrons on LHS, photons on RHS) are assumed to be uniformly distributed up to a maximum energy of 8 MeV. Broadening, due to bias in the energy selection, decreases with iteration number.

Several methods are used to speed the overall calculation including correlated sampling and reciprocity relationships. In this simplified problem, transformations (via rotations) of the energy scoring and boundary crossings of a single particle history enables concurrent transport of a particle over the full range of incident angles explored and ensures correlation with the reference calculation. A tally is maintained of boundaries crossed until all boundaries are crossed or the energy cut-off

( $\Delta$ ) is reached. Correlated sampling is used to improve the variance and takes the form [2],

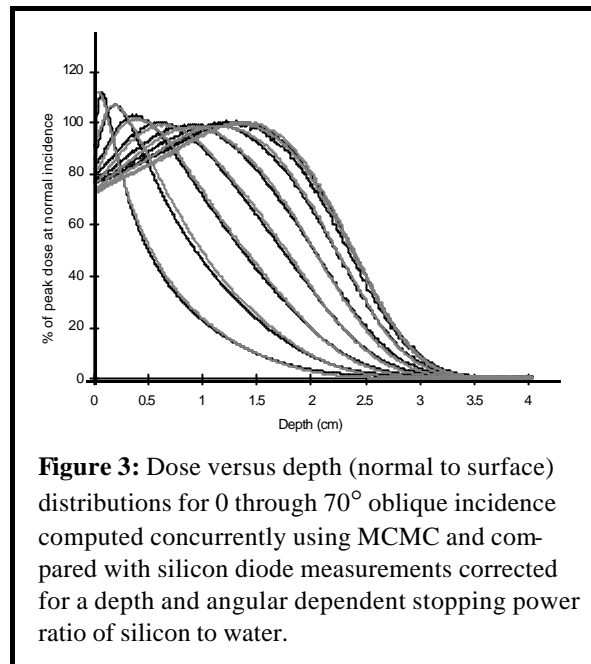
$$D = D_0 D'_i / D_i,$$

where  $D_0$  is a measured dose distribution and  $D_i$  and  $D'_i$  are  $i^{\text{th}}$  iterations of the MC computed dose distributions in the reference beam and patient, respectively. Finally, according to the reciprocity theorem, the energy deposited by a broad beam along a line normal to the surface of a semi-infinite phantom is equivalent to the energy deposited by a narrow beam in broad planar regions.

Depth-dose distributions for a range of obliquely directed beams ( $10^\circ$  to  $70^\circ$ ) have been subsequently computed. Calculations have been compared to measurements with a Scanditronix p-type silicon diode detector.

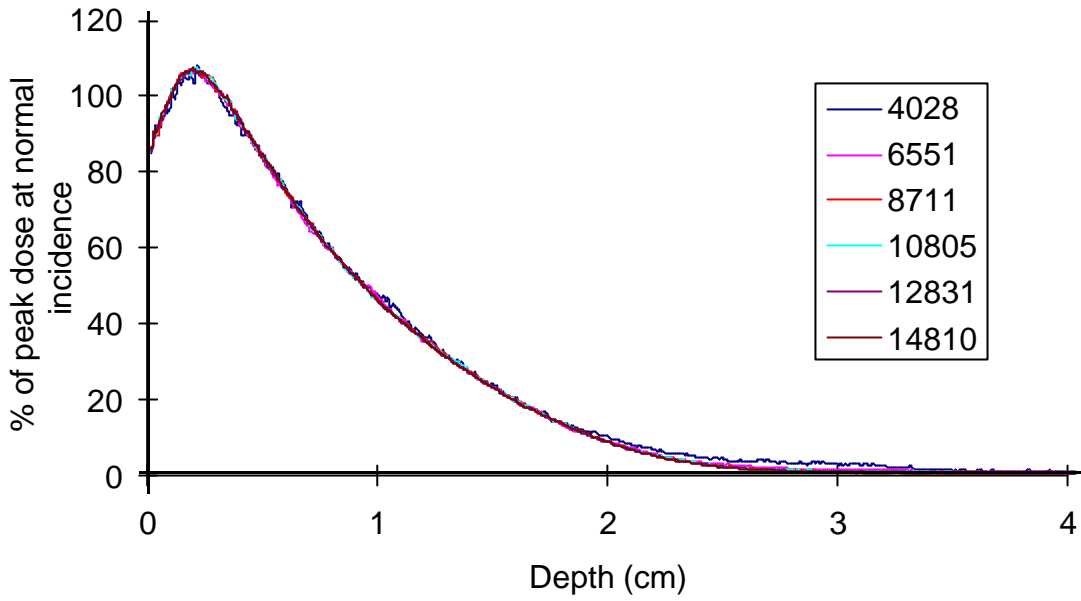
### Results and Discussion

Dose versus depth distributions have been computed concurrently for beams of normal through to  $70^\circ$  oblique incidence (figure 3) in 9.9 hours on a Sparc Ultra II. ( $\Delta = 100$  keV, ESTEPE=1%). Differences between the distributions can be noted but these are consistent with differences noted in the reference data measured at normal incidence (Figure 1) and are due to discrepancies in the responses of the two detectors. None-the-less, the simulation agrees with measurements to within 3% of the dose or 3 mm depth precision that is clinical requirement for radiotherapy planning. The calculations achieve a statistical accuracy of 0.495% (2 s.d.) of the peak dose. A surface dose enhancement for high angles of incidence which has been previously reported in the literature has been demonstrated by both methods. The effect is contrary to an expected decrease in surface dose from the dilution of fluence at a larger presenting surface such as is observed with photon beams. The enhancement is caused by increased scatter towards the surface and as such is not predicted by a depth scaling theorem.

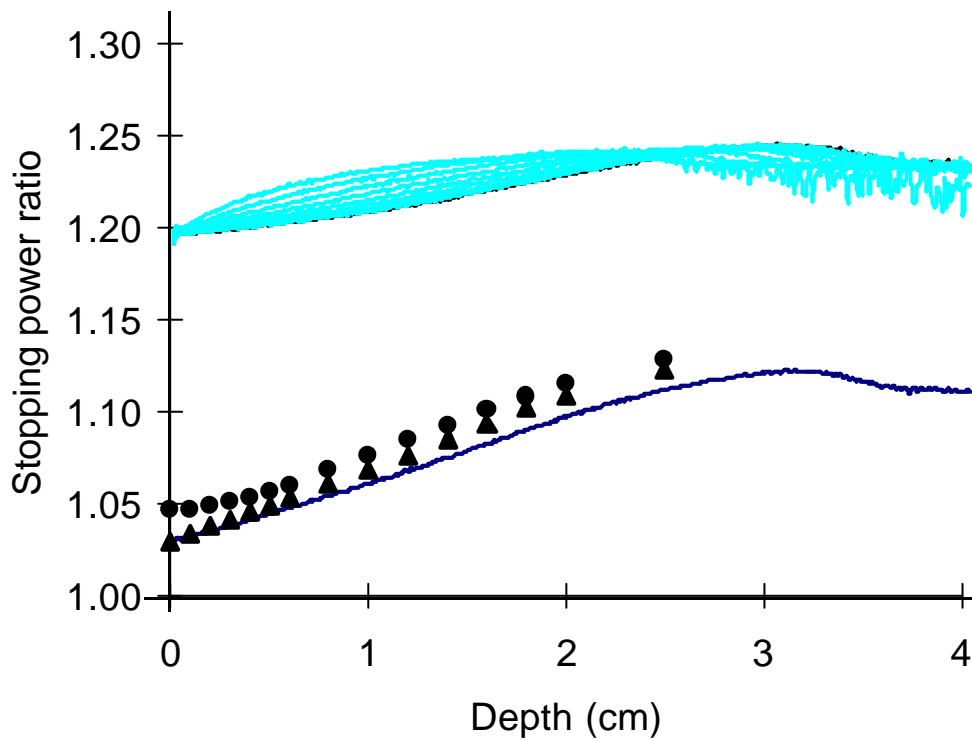


The dose distributions converge rapidly with iteration number (Figure 4). This result demonstrates that the broadening in the spectrum determined by MCMC as evidenced in Figure 2 is not necessarily reflected in the dose calculation particularly if there is a high degree of correlation between the reference geometry and the patient geometry or, as is the case here, that the energy deposition and transport is concurrent.

Mass-collision stopping power corrections for air to water with depth are within 1% of the tabulation in the IPEM 1997 protocol for electron dosimetry (Figure 5). The protocol, however, is based on MC calculations performed on monoenergetic beams. The ICRU (report 35, 1984) mass-collision stopping power ratios follow a similar trend to our results though 2% higher suggesting an error in the identification of the mean energy at the surface. The stopping-power corrections for diode to water have been computed for all angles demonstrates the power of an MCMC based algorithm to compute corrections for the dosimeter of choice as well as the dose distribution.



**Figure 4:** Dose distribution for a  $60^\circ$  obliquely incident beam as a function of iteration number which is incremented with each successful sample (a total of 10000 samples per batch). The variance at peak, post burn-in, is 0.495% (2SD).



**Figure 5:** The mass-collision stopping power ratio water:silicon as a function of the angle of incidence (top) and for water:air (bottom) compared with values interpolated for electron beams with a mean energy at the surface of 5.395 MeV from tables in ICRU-35 (circles) and the IPEM 1997 electron protocol (triangles).

## **Conclusion**

The successful application of the Markov chain Monte Carlo method for the determination of perturbed dose distributions from an unspecified radiation source has been demonstrated. The concurrent determination of dosimetric corrections enables more accurate commissioning and better dosimetry in non-reference circumstances. The technique contributes to ongoing research into the dosimetry of oblique electron beams used in total skin electron therapy.

## **References**

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